

On degrees of exclusion within and among systems**

*Ideas do not have to be correct in order to be good; it's
only necessary that, if they fail, they do so in an interesting way*

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ABSTRACT

The Color Exclusion Problem imposes *inter alia* that logic must look to conceptual arrangements in the world. This represents the collapse of what Wittgenstein held in his tractarian account of Logic (cf. 5.473 and 5.551). The kind of logical structure that we find inside elementary propositions in systems cannot be expressed in terms of truth-functionality, i.e., with logical products and sums. Moreover, since elementary propositions cannot be properly captured by the negation of a repeated item, the kind of exclusion within systems cannot be the contradiction. This means the necessary emergence of semantics in an alleged syntactical domain. We require then new tools to tackle non-neutral operators with restrictive scope and finer semantic exclusions. I introduce here, then, the notion of degrees of exclusion both internal and external (of second order) to a system. These degrees of exclusion may not be homogeneous, but must be infinite depending on the system of proposition that we are working with. Along the way, I advocate that we cannot reduce one system to another without distortions and that doing so is not necessary either.

Keywords: Logic; Exclusion; Contradiction; Contrariety; Truth-functionality; Systems.

RESUMO

O Problema da Exclusão das Cores impõe *inter alia* que a lógica deva olhar para arranjos conceituais no mundo. Isto representa o colapso do que Wittgenstein defendia na explicação tractariana da lógica (cf. 5.473 e 5.551). O tipo de estrutura lógica que nós encontramos dentro das proposições elementares não pode ser expresso em termos de verofuncionalidade, i.e., com produtos e somas lógicas. Além disso, uma vez que proposições elementares não podem ser capturadas adequadamente através da negação de um elemento repetido, o tipo de exclusão dentro de sistemas não pode ser a da contradição. Isto significa a emergência necessária da semântica em um domínio alegadamente sintático. Nós demandamos então novas ferramentas para expressar operadores não-neutros com escopos restritos e exclusões semânticas mais finas. Introduzo aqui, então, a noção de graus de exclusão interna e externa (de segunda ordem) a um sistema. Estes graus de exclusão podem não ser homogêneos, mas devem ser infinitos dependendo do sistema de proposições em que estivermos trabalhando. Ao longo do caminho, defendo que não podemos reduzir um sistema a outro sem distorções e que fazer isto é tampouco necessário.

Palavras-chave: Lógica; Exclusão; Contradição; Contrariedade; Verofuncionalidade; Sistemas.

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1 Introduction¹

“Wer nur einen Hammer hat, für den sieht jedes Problem wie ein Nagel aus”². This German proverb holds that for those who only have a hammer, every problem looks like a nail. Condensed but explicit in this maxim we may see the difficulties that the *Tractatus* faced when attempting to carry out a complete analysis of all legitimate propositions, or empirical ones, in terms of truth-functionality. Even generalities should be fully analyzed in these terms for they are also held as complex propositions. So they should be constructed “palpably” by truth-function means through their constituent elementary propositions (cf. 4.411). Indeed, according to the *Tractatus*, either a proposition is an elementary one or it must be able to be analyzed in terms of elementary propositions that define all the truth conditions of complex propositions (or of any proposition with any degree of logical complexity). In the best of the atomistic spirit nobility: if we have all elementary propositions, we would have the complete map of possible articulations of all complex propositions, i.e., a complete mapping of the comprehensive, absolute articulatory horizon of all facts, without surprises. That is, nothing new could be discovered in this maximal set of possibilities or logical space; just as we cannot find a new element in a system of coordinates or just as we cannot find new spatial points in the space, but only things set over this background³ (cf. Silva 2011, 2012 and 2013).

The *Tractatus* requires complex propositions being truth-functions of elementary propositions, an articulated aggregate representative of state of affairs, which by its names touches the reality. Following the suggestion from the above exegetical maxim, this kind of compositional analysis is the hammer to carry out the nailing, i.e., the complete analysis of all propositions. In this example, nothing is missing and nothing is left, because there are only hammers and nails suited to the task of nailing⁴. Nevertheless, when

¹ Here I use PB for *Philosophische Bemerkungen*, WWK for *Wittgenstein und der Wiener Kreis* and Some Remarks for *Some Remarks on Logical Form*. All the decimal numbers in the text come from the *Tractatus*.

² It seems that this *Sinnspruch* originally came from the Austrian psychotherapist and philosopher, Paul Watzlawick. It is a kind of popular sedimentation of Watzlawick’s constructivist theories in psychology.

³ For instance, we can find a hat in a room, but not a new spatial point in this room. Spatial points arise in our empirical propositions differently as spatial objects of our experience. The former set the description form of objects of our experience. Organized spatial points are necessary so we can speak of objects of experience. “Ein Raumpunkt stellt also eine Möglichkeit dar, nämlich die Möglichkeit der Lage eines Körpers relativ zu andern Körpern. Der Ausdruck dieser Möglichkeit ist der, dass der Satz, der diese Lage beschreibt, Sinn hat. Der Gesamtheit der Raumpunkte entspricht eine Gesamtheit von Möglichkeiten, also eine Klasse von sinnvollen Sätzen”. Or further: “Dann muss aber die Angabe eines Raumpunktes schon die Beziehung zu den andern Raumpunkten enthalten, und das heisst: Die Beziehungen zwischen den Raumpunkten sind intern. Wenn wir die Raumpunkte richtig einführen, so müssen wir sie mit einem Schlag samt allen ihren Beziehungen einführen”. (WWK, p. 215). Here we can see how Wittgenstein was engaged in transcendental arguments in this period of his thought and how his account of *Raumpunkte* follows in many senses the very account of the tractarian objects’ logical features. This is no accident (see Silva, 2012).

⁴ Often the ridiculous approach in Philosophy is analogous to the ridiculous situation of a scientist who faces a problem by dismissing reality for an *ad hoc* model which could indeed be bold, lean, attractive, but non-operative on many levels. Or the case of a government that attempts to dissolve the people because there

considering empirical propositions we note that they often contain generalities which are expressed nowadays in predicate calculus in terms of quantification, when a quantifier is analyzed, for example, as a predicate of a second order. The quantification of the *Tractatus* is made from logical products and sums, which lead to the largely metaphysical necessity that an elementary basis is fully defined and complete, always available and without empty references. Furthermore, we should demand that they always have a kind of implicit additional clause – a type of closing clause. A further proposition to complete the construction of quantification in the *Tractatus* seems always to be required: “and these are all disjunctions or conjunctions”, “and that’s all”, “and nothing else” etc.⁵

Yet, often when we deal with empirical propositions we need a more sensitive denial than the propositional one in order to differentiate what is really being denied within propositions. In a trivial example, we may affirm: “it is not the case that there is a black cat on the big table?” What is really being denied in this proposition? It is a predicate? It is a relationship? It is the instantiation of a predicate? (Is there no cat? Is there no table? Is it the relationship of “being on” which is false? Are the predicates “black” or “big” misapplied? Is it the table which is black or is it the cat that is big? Are neither of them black or big?...). If we hold that the only negation that we have is the propositional one, we are locally blind, that is, we cannot see what is being negated inside the proposition. In this case the strictly truth-functional tractarian analysis, headed by the propositional negation, seems to be not fully satisfactory.

However, these are local or peripheral issues. The two former “problems”, with the quantification and negation, can be harmless to the *Tractatus* if helped out by the metaphysical claims of the young Wittgenstein. That is, taking into account our image: the truth-functional hammer is sufficient for these shortcomings in quantification and in predicative denial, still coping with all the nails. The tractarian metaphysics is generous enough to supplement these “flaws”. However, the situation is dramatic if we take the problem of analysis of some common empirical propositions, such as ascriptions of color. For example, propositions like “this point is red” and “this same point is blue” exclude each other, if combined. This naturally leads one to think of them as not yet fully analyzed because they still contain logical complexity. Being

are many conflicts between them both. If our categorical system is poor, this should be discarded and not the reality. We have then to keep ourselves aware. We know that the form of the analysis determines (or even interferes or contaminates) the product of the analysis.

⁵ Actually, when we hold that all totalities in Wittgenstein’s tractarian period are indeed exhaustive, we do not need this closing clause. If we select one member of this totality, this will bring with him (*mitbringen*) all its connections with all other elements of this totality. In the discussions with the Vienna Circle, Wittgenstein very instructively attests: “Eine Klasse von wahren Sätzen wird in ganz anderer Weise begrenzt als eine Klasse von sinnvollen Sätzen. Im ersten Fall wird die Grenze durch die Erfahrung gezogen, im zweiten Fall durch die Syntax der Sprache. Die Erfahrung begrenzt die Sätze von äussen, die Syntax von innen. (p.213-214). For further discussion see Silva 2012 and Silva 2013.

consistent with our principles, we then try to give more time to the work of our truth-functional hammer. This spirit is clear in the tractarian 6.3751: the recurrent postponement of the task, as the secondary literature already shows (cf. Hintikka & Hintikka, 1986; Von Wright, 1996; Kienzler, 1997; Prado Neto, 2003; Marion, 1998). This kind of ascription leads indeed to the collision of two central tractarian theses: the logical independence of elementary propositions and the demand for complete analysis.

2 The hammer is not enough!

The demand for the full and unambiguous analysis of the language is simple enough to be proposed, but impossible to be implemented without difficulties. A crucial leitmotif for the tractarian project is that the thought or proposition is hidden or disguised by its grammatical form. In 4.002, Wittgenstein uses the same phrasing as Frege's *Der Gedanke*: "Die Sprache verkleidet den Gedanken". In his influential essay, Frege maintains: „Der Gedanke ist der Sinn eines Satzes, ohne damit behaupten zu wollen, dass der Sinn jedes Satzes ein Gedanke sei. Der an sich unsinnliche Gedanke kleidet sich in das sinnliche Gewand des Satzes und wird uns damit fassbarer. Wir sagen, der Satz drücke einen Gedanken aus" (p. 38-39).

The very idea that there is something logically relevant hidden in the language generates the natural demand of analysis. This represents the unique part of the *Tractatus* where Russell is praised. (cf. 4.0031). Ironically, this appraisal lasts until his official return to Philosophy (1929). It is indeed the whole image of language and logic which supports this appraisal which has to be dropped. The fall of the idea of something hidden to be brought to the surface by analysis represents the ultimate fall of the tractarian falling apart. This is shown in the entry "On Dogmatismus" in *WWK*, from 1931 (for further discussion, see Silva, 2012). The very idea that something is hidden is held then as something pernicious to the understanding of language. Furthermore, the search for prospecting the real logical form of a proposition already and invariably generates embarrassment when attempting to make this analysis in a perspicuous, unambiguous and comprehensive way. "There is one and only one complete analysis of the proposition", states Wittgenstein in 3.25. There should be, therefore, a kind of final and complete logical dismemberment of the propositions of ordinary language. In this breakdown, the final parts would somehow designate logical simple objects in reality, i.e., objects no longer susceptible to description. All linguistic complexity should be reduced to this atomic base. Rightly or wrongly, this requirement is intuitively compatible with the demands of a determined representation of the facts that make up the world.

In the *Tractatus* 6.3751, Wittgenstein symptomatically indicates that the problematic kind of exclusion of colors should be considered in the ascription

of different velocities to the same particle. However, we would still have an obvious exclusion since no particle can have simultaneously two different velocities. Hence we would have to continue the analysis. Nothing indicates that this exclusion may in fact be “sublimated” by a truth-functional analysis, especially because the conjunction does not work as an addition, as he admits later on (PB, p. 317). For example, when thinking about three meters, we cannot consider this length as “1 meter and 1 meter and 1 meter,” as this would simplistically be saying “1 meter”. Nor can we analyze 3 meters as “2 meters and 1 meter” as that would be absurd. Logical products cannot express gradations or degrees. This problem points out greater difficulties which indicate the need to consider different kind of conjunctions for different kinds of articulations in systems of propositions [*Satzsysteme*]. For example, there is a need for a combination which regards the number of occurrences of variables for the context of measurements. Similarly, when considering colors, there is a need for a combination which allows mixtures for some colors, but not for others, as in, respectively, of blue and red, and of blue and orange. Logical connectives should be contextually sensitive and tested *ad hoc* for applicability (see PB’s paragraph 83 and discussions in Silva 2011, 2012).

This investigation shows that neither of these problems with gradations (be they measurements or colors) can be regarded as a “nail” and that the old “hammer” is not really enough to handle the analysis of all empirical propositions. We could perhaps have something like an embarrassing exit and deny the empirical status of propositions that involve gradations in order to keep with the truth-functionality of all propositions. Translating this exit to our guiding principles, we would say something like: “my hammer is still excellent, but your nails are not actually nails”. This would be equivalent of saying this problem is indeed no problem. We could also adopt a more pragmatic exit reviewing our parameters or creating new ones from new perceived problems. In this vein, one could say: “I give up either the complete analysis, or the independence of elementary propositions, allowing, then, that they do exclude or imply each other in organized systems (*Satzsystem*)”. This strategy is more *ad hoc* and irreversibly induces the revision of the *Tractatus*, since Wittgenstein assumes the need to start looking into the propositions. For it, the normative appeal of Logic has to be mitigated. The current language before being regimented by the authoritative tractarian thread to avoid philosophical nonsense, now has shown to it a robust deficiency and waits a rearrangement of the project.

3 Introducing the notion of degrees of exclusion

In the *Tractatus*, if p and q are elementary propositions they should be independent, because the concatenation of them does not generate contradictions, because there is no denial or repetition in elementary propositions. But if p and q

belong to the same system, such as a length measuring system or the assignment of color to visual points, we can have exclusions without repetition or negation. We can pose here numerous questions, such as: In a system of color ascription, does something being green in some sense deny it from being red? Does an object being square prevent it from being round in a classification system of geometric figures? In a biological taxonomy, does "being a lion" bar something from "being a leopard"? Does "to be a lion" disallow "to be a domestic cat" more than "to be a leopard"? Does "being green" exclude something from "being red" more than it obstructs it from "being yellow"? Whatever their answers are, what these questions show us is that there are "negations" (or exclusions), or incompatibilities in our daily lives that appear not to be brought about by repetition and denial, and which seem to be definitely distinguishable from the kind of exclusion by contradiction. Thus, these exclusions appear to be utterly beyond the power of analysis required by bipolarity and carried out by the prominence of the truth-functionality. If the base is meaningful, this meaningfulness does not guarantee the meaningfulness of the complex strictly generate from this basis. It is not just "p" and "q" and their truth conditions which are relevant for the composition of "p and q", but the kind of articulation between both. Moreover, these exclusions bear infinite degrees of strength and proximities (*Nähe*). As Wittgenstein himself speculates in the paragraph 218 of PB, regarding the color metric:

Man kann nun unmittelbar Farben als Mischungen von rot, grün, blau, gelb, schwarz, und weiß erkennen. Dabei ist Farbe immer color, nie pigmentum, nie Licht, nie Vorgang auf oder in der Netzhaut etc. Man kann auch sehen, dass die eine Farbe rötlicher ist als die andere oder weißlicher etc. Aber kann ich eine Metrik der Farben etwa Bezug auf ihren Gehalt an Rot in der Mitte zwischen zwei anderen Farben steht? *Es scheint jedenfalls einen Sinn zu haben zu sagen, die eine Farbe steht einer andern in dieser Beziehung näher als einer dritten* (p. 273, my italics).

For example, it would be enough to take a point in the mosaic or in the continuum of colors and systematically compare this point to other points (perhaps spatially) closer and more distant in this system. If, in fact, colors form a dense system, i.e., for every two points in the range of colors we have a differentiable point between them, we will have, following this argument, infinite degrees of strength of exclusion.

The complexity of the architecture and organization of colors imposes sophistication on the tractarian logic that it cannot provide. In other words, the color ascription shows that there are, at least, some empirical propositions that have many (potentially infinite) corresponding negatives, as in the case of the ascription of degrees to empirical qualities. At least in these cases, this happens because we have several other propositions that are partially, but not "totally", outside of them, as argued in 5.513. We have two crucial aspects here: There are some propositions which are fully meaningful as they admit many possible and

meaningful negations and these negations are not completely outside of them. This negation has to be another one than the classical one, since its scope is always restrictive. In these cases, they all belong together to the same system or conceptual background, which builds up this logical affinity or familiarity. If I say that a table is green, this necessarily excludes it from being red or yellow... In the same way, if I say that the table is 3 meters long that means that it isn't (nor can it be) 4 meters nor 2 meters long... There is this kind of map of exclusions in all component elements of any system of classification as well, i.e., wherever we have a multiplicity categorically organized with more than two alternatives.

A pattern of exclusion different than the contradiction and above the third excluded principle emerges here. For, if I say that a certain animal in front of me is a lion, this excludes this animal from being a turtle, or a human being, and it is also excluded from being a domestic cat or a leopard. It is hard not seeing necessity in all these exclusions, if we do understand how the classification works. This is the same with trichotomies: as is the case when, for example, we are informed that a football match did not end in a draw and we can say that one of the two teams won or lost. Here for the proposition "the game ended up in a draw" there is not only one possible negative but two, for each team either won or lost. And these two remaining alternatives are not totally outside the first proposition as they share the same conceptual background or system of familiarities, in this example, a league table of soccer results.

The elements of these conceptual systems seem to be grouped in categories that exclude elements of other categories. But within these categories, components also differ from each other in a way that the presence or the identification of one automatically excludes the other. In this way it makes sense to develop the idea that there is not just one kind of exclusion, but infinite kinds of exclusions, with different degrees or forces. In an organized group or system⁶ that consists of, say, (green, yellow, blue,

⁶ Here I am adopting an intuitive sense of categorical organization, commonly adopted by linguists to deal with paradoxes of exclusions – a sort of micro-system. A micro-system that can give us lessons on complex systems in fact, as the one of numbers or colors. My objective with this is to show the naturalness of contrariety exclusions in everyday life. This logic pattern arises in so many situations, that it may suggest that it is a necessary feature in our daily sense for exclusion: when we have more than two alternatives in a system. I am not interested here in more sophisticated or abstract kinds of groups organized by logical characteristics as (rot, blau, gelb, blauer), where "blauer" is the element that does not belong to the group because it is a binary relation and not a simple predicate. Correlatively, we could build a group with the following terms (blauer, grösser, länger, schneller, rot), where "rot" has to be detached because it is not a binary relation. We could represent a group of first-order predicates, without representing exclusions by contrariety, as (rot, bunt, gross, schön). Such a group would allow the four elements here to be simultaneously ascribed to an object in the field of discourse. What is interesting here is that if we place "blau" into this group, we would then have again automatically the phenomenon of exclusion by contrariety between "rot" and "blau". This is because an object, regardless of what it is, in a domain of discourse, regardless of what it is, cannot have these two predicates simultaneously. This becomes clearer if we think about the truth-functional conjunction. We could think of a "color" operator to simulate the impossibility of conjuncting blue and red, but allowing the mixture of both in the form of violet. This operator "color" does not necessarily need to match the additive conjunction of arithmetic. After all, white does not mean the union of every color, in the same way that "three" means the junction of three units. We will return to this issue.

circular⁷) we can easily identify which element is the “outsider”, the “alien”. However, we also know that if one of these remaining qualities is assigned to an object in the visual field, the other qualities will automatically or necessarily be excluded. Logically excluded, I say. In a group or system that consists of (4 meters, 3 meters, 5 meters, 4°Celsius.) again we can identify, or exclude the alien component, and again we have to exclude elements of the same class, if a particular quality is ascribed to an object. Take, for example, a group or system that consists of animals (e.g. a group of animals made up of a lion, a leopard, a domestic cat and a turtle) we can identify, isolate, or exclude one as the “categorical alien” (in this example the non-feline turtle). And if we identify something as being in fact one of the leftover components, the others will be necessarily excluded. And this exclusion, I say, is also logical, in the sense it is also necessary.

All the components grouped into a system (or group) are obviously cognates as they exclude components of other groups, but they also are excluded within their own group. This is the fact and the problem. There is a paradox here with groups which is investigated by linguists, as is clear in the work of Dany Jaspers (cf. 2005 and 2011). Even when elements are organized into a group based on affinities or familiarities, and excluding other elements of other groups, they also exclude elements *within* their own group. We can easily think of levels or degrees of strength in the exclusion of external components of other categories and in the exclusion of internal elements. These exclusions, the internal and the external, must not be uniform. All exclusions here are not effectuated by contradiction, and nor do they admit being reduced in terms of contradictions. For example, a system or group consisting of a lion, a leopard, a domestic cat, and the number seven logically seems to exclude the “7” more intensely than the way the turtle is excluded from a group constituted only by animals, such as a group consisting of a lion, a leopard, a domestic cat, and a turtle. This is because the “number 7” is even more intensely foreign to the group of felines than a turtle would be. But the turtle would be more strongly excluded from this group than a dog would be, if, instead of a turtle, a dog were added amongst the felines. As these examples show, the exclusion of foreign elements clearly accepts variations of degree and strength.

In the case of exclusion within the system we have again this logical phenomenon. Rather more subtly, when ascribing a group component to an object in the world, other components are necessarily, automatically, or

⁷ Of course in a system with three elements, we can do have some ambiguity about the alien element to be discarded, as in (bear, monkey, banana) or in (blood, blue, red). To avoid this kind of problem I am adopting here systems with at least four items. It is interesting to note that to restrict the explicite cardinality of such systems does not prevent us of a (implicite) exposition of alternatives, since, at least, some elements cannot be understood alone or isolated. They do bring with them necessarily numerous other elements, if we do understand the scheme of exclusion in which they are built.

logically excluded. Apparently we may have the same strength of exclusion homogeneously applied to all member of a system, regardless of whichever system we are operating in. Here, however, there is also a variation in the strength of internal exclusion, as when attributing a degree of temperature, a volume or length to some empirical element, or a color to a visual point. This is precisely the case when we work on taxonomic systems, as we have seen, which in no way appear to belong to logic but also involve exclusions, as in the case of the group of felines. If something is a lion, it cannot be a domestic cat, a leopard, a jaguar, or a panther... "Lion" excludes "domestic cat" more strongly than it does "leopard", as lions and leopards are both big cats found in the wild while domestic cats are not. Similarly, if a point is blue, it is not green, nor is it purple or orange... Here blue excludes orange more strongly than it excludes green and purple because blue enters in the composition of both green and purple. All exclusions presented here seem to be automatic, necessary and logical, but they are not truth-functional. The sense and the truth of the complex do not depend strictly only on its present parts or elements. These logical exclusions cannot be represented in these terms, because they belong to some propositions that explode in many (in some cases, in infinite) alternatives when negated, always in relation to other propositions within the same system. The tractarian bipolarity could never express this explosion in terms of concatenation and not-concatenation of simple objects.

We could then also think of the interaction between systems or organizations and systems that include or exclude others. One can hold these relations as belonging to a second order within which exclusions and implications operate. These relations are not set among elements within a system but among systems themselves. This would also seem to enable us to organize systems within systems, so that we can exclude, with objective accuracy, the alien category or system. For example, in the case of a system consisting of countries, cities, continents and numbers, one would exclude the system of numbers. We could even try to find structural equivalences or isomorphisms between systems, which at first glance seem not to belong to the same second order system, nor to even have anything in common with one another. For example, the second order systems consisting of colors, numbers, and musical notes systems. Such research can reveal surprising structural equivalence between categorical systems which are completely different, perhaps forcing us to rearrange our knowledge. This makes Dany Jaspers' approach, although quite ambitious, fascinating (Jaspers, 2011).

We may add up to our investigation that this kind of generous mosaic of internal or external exclusions within a system and between systems must not by any means be necessarily homogenous or uniform. The notion of familiarity or *Verwandschaft* seems to play a crucial role in the organization of *Satzsysteme*. For example, in a system of oppositions of colors, green appears to exclude red

more strongly than it excludes yellow. And, in a system of geometric figures, a triangle and a square have more in common with each other than they do with a sphere. Panthers and leopards are closer to each other than to domestic cats, despite all of them being felines, and necessarily excluding others. If we could organize these degrees of strength on a scale of exclusion, we would be compelled to expect, to some extent, that there is a continuum of degrees of logical exclusions: from the most “pure” and radical logical exclusion, in a narrow sense, like the contradictory exclusions to more empirical exclusions, as in taxonomic systems which are contingent, non-definitive and historically dynamic, yet logical, because within them the exclusions are all necessary. In a certain sense, logic should not only look at the empirical facts. Rather, logic should touch facts and even mix itself with them. Logic should inform the world organization. This seems to be a natural development of the idea that the logical form should be incorporated in ontological elements (cf. 2.01-2.0141).

4 Several kinds of conjunction

It is doubtful that a purely combinatorial and neutral pattern of truth-functional can capture this kind of subtlety of organization in systems and among systems. In all these cases, although elementary propositions make sense separately, the junction of these may simply not be allowed. The truth table notation, which incorporates the tractarian picture of logic, entails a procedure of symbolic expression which is tentatively neutral, complete, mechanical and combinatorial (cf. Silva 2012). However this is not suitable for capturing finer exclusions because some combinations should be *a priori* excluded. Some truth table lines have to be mutilated, in order to provide a correct representation of the possible articulations of the complex to be represented (cf. von Wright, 1996). No generalization of this mutilation of some lines for all systems is to be expected. For example, depending on the kind of conjunction that we are using, we could match blue and red to obtain purple, but not orange and purple. Or, while it can be said that “3 is 1 plus 1 plus 1” this same “plus” cannot be employed to show that purple is “red plus blue”, maybe due to the same reason that white must not be either the addition of all colors. So does “understanding white” mean understanding the presence of all colors, in the same way that understanding the number 3 means knowing how to join up three units? Nobody has to know this fact about white to correctly use the predicate “being white”. This is not to be expected with the predicate “being three”, for to use that predicate adequately one has to know how to decompose it in terms of “being one” and of “being two”. Moreover, what would it mean to “match” or to “conjoin” a leopard with a lion? While it could be that within the domestic cat species there are some breeds that are “matches” of other two different breeds, but there can also be some that cannot be “matched”.

The meaningfulness of some elementary propositions does not guarantee in all systems the meaningfulness of the complex propositions done by their articulations. Some articulations have to be *ad hoc* forbidden in the system and then in the former notation.

There are more logical connections than the tractarian logic, with its tautologies, contradictions, logical products and sums and truth-functionality, can express. Color systems and numbers allow compositions that cannot be made with the traditional conjunction – a truth-functional apparatus that would only generate truth-functional exclusions or implications, without the sensitivity for a different system. However, adding up numbers does not mean mixing up colors. While we can indefinitely sum up units to arrive at a certain number, it is hard to believe that we can continuously add the same color to itself or even add different colors to each other *ad infinitum*. Moreover, what would be a unity of a color added up to another? A blue point can be dark blue in a certain visual context and light blue in another, while the “number two” will always be a prime number in any context. To mix up colors does not mean adding up colors. “Orange” is not “green + red” as much as “3” is “2 + 1”. “Mixing up” yellow and blue by the truth-functional conjunction seems to be patently absurd, but somehow, “yellow + blue” seems to make sense in generating green. However, when we say “green” we often do not mean “yellow and blue”, where “and” is a truth-functional copulation, and, in the strictest sense, not “yellow + blue.” For, while we can make a darker green by adding more blue to the composition with yellow, we cannot make a stronger 3 by adding more 2 in a composition with 1. Another problem: Blue can be mixed up with red, and green can also be mixed with red. But the resultant colors: purple and orange respectively, cannot really be mixed up with another, although in a certain sense both come from red. The color system needs its own operator for composition, different from the conjunction of the propositional calculus and from the addition of natural numbers. As the truth-functional conjunction will not generate “mixture” in the case of colors or “addition” in the case of numbers, for this we would need other kind of “conjunctions”. This means that we cannot reduce one system to another without distortions and that, we do not need to do so, since the grammar of different systems should be autonomous. This represent an alternative view to the one held by Cutler (2009).

Logical operators have to be more sensitive even if we lose the truth-functionality. We have here the opportunity to postulate the existence of at least three different types of combination or conjunction: a summative one, which counts the occurrence of elements, important to the mathematical context; a truth-functional one, which does not count occurrence, important for propositional logic; and a “color” one, for expressing possible combinations as “green and yellow” or “red and blue” but not “purple and orange.” In a certain sense, a visual point can indeed have two colors, depending on the colors that

are to be mixed up there. For example, a shirt can be indeed red and blue, if here we are using the “and” to mean the mixing of colors in color system. This shirt would be indeed purple in this example. We could then even assume here that for each propositional system we may have a kind of “conjunction” to express its own possible and impossible logical links via its peculiar kinship. Or in other words, we would need a special sensitive conjunction for each system for capturing its logical multiplicity. Recognizing new problems implies sharpening new tools.

5 We must look into elementary propositions and at the facts of the world

For all possible counterexamples to truth functionality the strict tractarian strategy would be the same: do not deny the meaning of these empirical propositions, but do indicate that their analysis is not over yet, because we still have a logical complexity or logical dependency between their constituents requiring further analysis. We can then legitimately ask ourselves: Are there no logical constructions but truth-functional ones? The tractarian response: “Of course not! If anything points to this, the analysis must then continue to the end! Or, in the case that something has apparently gone wrong with the analysis, keep on analyzing it, but in a proper way!” This would be a promissory note that could never be truly paid. The curse of the tractarian project is always having to indefinitely postpone its end.

The compositionality marked by truth-functionality does not capture all possible cases of empirical propositions. In this sense, it postpones the problem of conceptual linkage, but does not solve it. In fact, the *Tractatus* seems to have bet too much on truth-functionality and its corresponding truth table notation, and seems to have missed the point that some required dependencies (such as the implication and the exclusion) can be seen in the conceptual relations *within* propositions, among its components. The “within” here is crucial: it is not enough to get elementary propositions, but we also have to analyze their elementary components. We should look into the proposition, to its logical construction, to the logical complexity of its members.

Considered rigorously, the *Color Exclusion Problem* represents *prima facie* a challenge to the logic based on truth-functionality, and not to the tractarian account of Mathematics, Ethics, or the Philosophy of Science. The problem is ultimately with its image of logic and not with numbers or ethics and esthetics. Wittgenstein himself seems to acknowledge this interpretation in § 76 of the PB:

Man könnte sagen, die Farben haben zueinander eine elementare Verwandtschaft. Das lässt es erscheinen, als könne innerhalb des

Elementarsatzes eine Konstruktion möglich sein. D.h., *als gäbe es eine logische Konstruktion, die nicht mit Hilfe der Wahrheitsfunktionen arbeitet*. Nun aber scheint es außerdem, dass diese Konstruktionen eine Wirkung auf das logische Folgen eines Satzes aus einem anderem haben. Denn wenn verschiedene Grade einander ausschließen, so folgt aus dem Vorhandensein des einen, dass der andere nicht vorhanden ist. Dann können zwei Elementarsätze einander widersprechen. (p.106, my italics).

These types of relationships or constructions of the components of elementary propositions would not be formal, if we think of formality collapsing with truth-functionality. These links are not expressible by the truth-functionality of common logical operators. Or, in the *Tractatus*, by the *NOR* operator (cf. 6), the combined denial of an elementary basis of propositions, by the truth-functional completeness. The sense of the complexity is not derived from its components' sense. We have to presuppose more: the whole system in which these components are necessarily embedded. This leads us to believe that the tractarian logic of tautologies and contradictions is powerful, but rough. It may be rough, because it is too powerful! This lack of expressiveness means that the tractarian logic would necessarily begin to incorporate some points usually taken as extra-logical. Logic collides with its application. Logic has at last to look to the world to be executed properly. This quote taken from *Some Remarks* appears like an echo over this period: "And it would be surprising if the actual phenomena had nothing more to teach us about their structure" (p.164). Logic has begun to appeal to intuitive empirical features. Where in the (arrogant) neutrality of the *Tractatus* could we expect an appeal to the ordinary or current language such as that in the following passage?

Every one of us knows that in ordinary life. If someone asks us "What is the temperature outside?" and we said "Eighty degrees", and now he was to ask us again, "And is it ninety degrees?" "We should answer", "I told you it was eighty". We take the statement of a degree (of a temperature, for instance) to be a complete description which needs no supplementation. (Some Remarks, p.167).

This is opposed to the revealing passage in parenthesis in the *Tractatus*: „[...] (Und wenn wir in die Lage kommen, ein solches Problem [a logical one] durch Ansehen der Welt beantworten zu müssen, so zeigt dies, dass wir auf grundfalscher Fährte sind)" 5.551. In this way, *Some Remarks* teaches us that, in contrast to Wittgenstein's former vision, we have to take this "wrong" path. There is no proper language analysis, with or without a proper notation, without taking this wrong way: We have to look at the world in order to do Logic.

Conclusion

If we do hold that every proposition of our daily use expresses at least one empirical quality and that all empirical quality must be able to vary in degrees, we can see an all-encompassing problem to a theory of meaning that is based solely on truth-functionality. By means of logical product we cannot express addition of degrees. There are more logical oppositions and exclusions among empirical propositions than the logic of tautologies and contradictions of the *Tractatus* can express. From 1913 onwards, this is the picture of logic that Wittgenstein thought definitive:

Ich dasjenige, was ich in meinem letzten Brief über Logik schrieb, noch einmal in anderer Weise wiederholen: Alle Sätze der Logik sind Verallgemeinerungen von Tautologien und alle Verallgemeinerungen von Tautologien sind Sätze der Logik. Andere logische Sätze gibt es nicht. (Dies halte ich für definitiv). Letter to Russell from Norway, 1913 (Tagebücher 14-16 p. 127).

This combinatorial, neutral, abstract logic – a hallmark of the *Tractatus* – cannot express or carry out all empirical propositions. Some exclusions are simply opaque to the tractarian instruments of analysis – instruments which, although very potent, are too abstract and not sensitive enough to express finer exclusions and implications. The tractarian logic cannot express the difference between contrary and contradictory propositions. The former cannot be reduced or analyzed in terms of contradictory propositions. The former are more refined, because they represent an opposition in which two propositions cannot be true together although they can be false together. Not all exclusions are the kind of contradictions that can be well captured by the notation of truth tables. Some look extremely empirical. “Two things cannot occupy the same point”, or “at one point there cannot be two things”. Or, in an altogether contingent taxonomic system, if one affirms that an animal is a dog, it cannot (automatically, eo ipso, logically, or through necessity) be a domestic cat, or a horse or human being... Similarly, if a point in the visual field is blue, it cannot automatically be red, green, yellow ... If the temperature today is 22°C it is not, nor can it simultaneously be, 21°, 19° or 23°... And if a table is 3 meters long, it is not 4, 5 or even 2 meters long... And in a football match, if a team did not lose or draw a game, it must have won. This type of exclusion is internal to a category or a system of propositions: internal, that is, to a group organized by the internal resemblance between its elements, which allows many alternatives and not just two in the case of the tractarian bipolarity. In most of the examples given above the use of reticence precisely points out a context of numerous, if not infinite alternatives, depending on which system(s) we are operating or working within.

Even mistakes within a philosophical system can generate good by-products, because they bring up efforts which show some crucial characteristics of a domain. Sometimes to look for solutions to some problems are more seminal than to find them. The Color Exclusion Problem seems to throw logic into the world. That is, it seems to make logic less neutral and abstract. This imposes logic to the world with empirical exclusions and incompatibilities, but these are yet somehow logical ones. This fact allows us to postulate the very existence (or possibility of construction) of an infinite number of logical exclusion degrees, not to be expressed by the truth-functional paradigm. The truth functional hammer has shown itself to be too rough or even essentially inadequate to account for all problems. Not everything is a “nail”. Thus, either the problem is with the complete analysis, or with this kind of proposition which would not be total empirical or total logical. Or, by following our analogy, the problem would be either with the hammer or with the other kinds of nails. Going a bit further, by the end of 1931 Wittgenstein reached the conclusion that the problem was with both the hammer *and* the nails. The problem is largely with this way of trying to regulate language, this attempt to find an instrument or general method for prospecting and bringing to light a hidden essence. The problems arise with this kind of regulation of language through linear rules which produce more distortions than solutions. In 1931, the very idea of complete analysis was to be abandoned: there was no longer to be an idea of a hammer suitable for all nails. It was found that the problem was less about having a good hammer for all nails than it was about encompassing everything could be taken as a nail. Neither the whole nor a part of our language must be exclusively pictorial. Not all linguistic contexts need linguistic or ontological atoms, or a homogeneous and precise association between them. We do not need a complete analysis, or even any logical analysis, in our language to guarantee the determination of the propositional sense. Indeed, Wittgenstein recognized that in some contexts sense need not be fully determined, and that, at some times and in some contexts and language activities, there are some essential indeterminacies.

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